

Formulas, units & physical relationships

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MATTER AND RADIATION

$$E = h f = h c / \lambda$$

Exact for a photon in vacuum. E J; f Hz; λ m; h J s; c m/s.

$$I = I_0 e^{-\mu x}$$

Ideal narrow, monoenergetic beam in uniform material; μ and x use reciprocal units.

$$\text{HVL} = \ln(2) / \mu$$

Half-value layer, same monoenergetic narrow-beam assumptions as above.

$$E(\text{keV}) \approx 1.240 / \lambda(\text{nm})$$

Unit conversion for photon energy in vacuum; rounded constant.

$$\text{Flux or fluence rate} \propto 1 / r^2$$

Point isotropic source in free space; neglect attenuation and scatter.

RADIOGRAPHY AND FLUOROSCOPY

$$\text{mAs} = \text{tube current (mA)} \times \text{exposure time (s)}$$

Definition. Dose output often rises with mAs when all other parameters are fixed.

$$M = \text{SID} / \text{SOD}$$

Ideal geometric magnification: source-image / source-object distance.

$$U_g \approx f \times \text{OID} / \text{SOD}$$

Geometric unsharpness for a finite focus f ; OID object-image distance.

$$\text{Relative quantum noise} \approx 1 / \sqrt{N}$$

Poisson-limited detection: more detected quanta improve signal-to-noise.

$$\text{Output at detector} \approx \text{mAs} / r^2$$

Fixed spectrum, filtration and geometry; ignores attenuation and AEC responses.

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RADIONUCLIDE IMAGING

$$A(t) = A_0 e^{-\lambda t}$$

Radioactive decay law; activity A is in Bq (decays per second).

$$T_{1/2} = \ln(2) / \lambda$$

Physical half-life T in s, h or d if decay constant uses reciprocal unit.

$$1 / T_{\text{eff}} = 1 / T_{\text{phys}} + 1 / T_{\text{bio}}$$

Applies to independent monoexponential physical decay and biological clearance.

$$N \approx \text{efficiency} \times A \times \text{counting time}$$

Approximate for nearly constant activity, fixed geometry and efficiency.

$$\sigma(N) = \sqrt{N}$$

Poisson count standard deviation; relative uncertainty is $1 / \sqrt{N}$.

RADIATION SAFETY

$$D = \text{deposited energy} / \text{mass}$$

Absorbed dose: gray (Gy) = J/kg. An average over the specified mass.

$$H_T = \sum_R w_R \times D_{T,R}$$

Equivalent dose, in Sv; radiation weighting factors apply by definition.

$$E = \sum_T w_T \times H_T$$

Effective dose, in Sv; reference-person protection quantity, not individual risk.

$$\text{Rate at distance approximately} \propto 1 / r^2$$

Useful point-source limit; barriers, scatter and finite sources alter it.

$$\text{Transmission} = e^{-\mu x}$$

Ideal narrow monoenergetic beam; diagnostic broad-beam shielding needs corrections.

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COMPUTED TOMOGRAPHY

$$HU = 1000 \times (\mu_t - \mu_w) / \mu_w$$

Nominal CT number from reconstructed effective attenuation, water reference.

$$CTDI_w = (CTDI_{\text{centre}} + 2 \times CTDI_{\text{periphery}}) / 3$$

Weighted scanner output index for standard CTDI phantom measurements.

$$CTDI_{\text{vol}} = CTDI_w / \text{pitch}$$

Helical convention; for contiguous axial scans $CTDI_{\text{vol}} = CTDI_w$.

$$DLP = CTDI_{\text{vol}} \times \text{scan length}$$

mGy cm if $CTDI_{\text{vol}}$ in mGy and z-axis coverage in cm.

$$E(\text{mSv}) \approx k \times DLP(\text{mGy cm})$$

Population/region dependent conversion; not an individual organ-dose estimate.

$$\text{Noise approximately} \propto 1 / \sqrt{(\text{mAs})}$$

Assumes fixed kVp, patient, detector response and reconstruction.

MAGNETIC RESONANCE IMAGING

$$f_0 = (\gamma / 2\pi) \times B_0$$

Larmor frequency Hz if gamma is angular gyromagnetic ratio rad/s/T.

$$^1\text{H } f_0 \approx 42.58 \text{ MHz/T} \times B_0$$

Approximate proton gyromagnetic ratio in a static field B_0 (tesla).

$$M_z(t) = M_0 \times (1 - e^{-t/T_1})$$

Recovery after complete saturation: ideal simple T_1 model.

$$\text{SE signal} \sim \rho (1 - e^{-TR/T_1}) e^{-TE/T_2}$$

Simplified spin-echo weighting; ignores details of excitation and steady state.

$$FOV = 1 / \Delta k$$

Cartesian spatial-frequency sampling with k in cycles per metre.

$$\text{Scan time} \approx TR \times N_{\text{phase}} \times \text{NEX}$$

One phase-encode line per TR ; echo trains, parallel imaging change timing.

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ULTRASOUND

$$c = f \times \lambda$$

Wave speed m/s, frequency Hz, wavelength m. Soft tissue: $c \sim 1540$ m/s.

$$\text{Depth} = c \times \text{round-trip time} / 2$$

Pulse-echo ranging assumes fixed sound speed in tissue.

$$\Delta f = (2 f_0 v \cos(\theta)) / c$$

Backscattered Doppler shift; theta angle between beam and flow direction.

$$|\text{Doppler shift}| \text{ unaliased} < \text{PRF} / 2$$

Nyquist limit in sampled pulsed Doppler; PRF in Hz.

$$\text{Axial resolution} \approx \text{spatial pulse length} / 2$$

SPL = cycles x wavelength; short pulses improve axial resolution.

$$\text{Maximum range} \approx c / (2 \text{ PRF})$$

Simple single-pulse echo sampling limit; depth and PRF trade off.

UNITS AND COMMON TRAPS

$$1 \text{ Bq} = 1 \text{ transformation} / \text{second}$$

Activity, not absorbed dose; detector counts also depend on efficiency.

$$1 \text{ Gy} = 1 \text{ J} / \text{kg}$$

Absorbed energy per unit mass. Not automatically equal numerically to Sv.

$$1 \text{ Sv} = 1 \text{ J} / \text{kg} \text{ with weighting}$$

Equivalent/effective protection quantities have different interpretations.

$$1 \text{ keV} = 1.60218 \times 10^{-16} \text{ J}$$

Energy conversion; $1 \text{ MHz} = 10^6 \text{ Hz}$ and $1 \text{ cm} = 10^{-2} \text{ m}$.

Always write the assumption beside a relationship

Fixed technique? Monoenergetic? Point source? The answer can change.